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Autocorrelation Measurements in the Near Wake of Square Cylinders in Turbulent Free Streams

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statistical ripples, the results of this process agree with those obtained from Eq. (5).

The following results were obtained from a problem in which the kinematic viscosity $\nu = 0.01$ and $a = 1.0$ in the smoothing function, corresponding to a Reynolds number of about 300.

Figure 1 shows energy decay curves, $U(0, t)$ vs t, curves I and II corresponding to results from Kraichnan's and Burgers' equations, respectively. Curve III shows the energy history if the transfer term were zero and only viscous dissipation acted. It can be seen that at early times the approximation is good, while by the time the energy has dropped to 10% of its original value, the error has increased to 30%.

Figure 2 shows normalized correlations obtained when the energy has dropped to approximately 20% of its original value ($t = 1.6$); again, curves I and II give Kraichnan's and Burgers' results, respectively.

A most interesting comparison for $t = 1.6$ appears in Fig. 3, in which curve I represents

$$T(r, t) = \frac{\partial}{\partial r}\,[U(0, t) - U(r, t)]^2$$

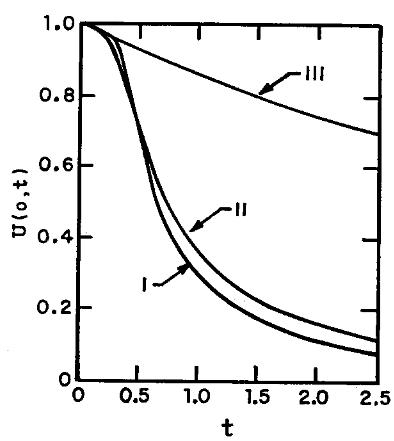

FIG. 1. Energy decay curves $U(0, t)$, obtained by integration of, I: Kraichnan's equation; II: Burgers' equation; III: Kraichnan's equation if the transfer term is zero (i.e., only viscous dissipation acting).

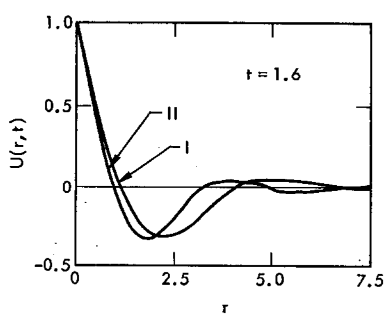

FIG. 2. Normalized velocity correlations $U(r, t)$ obtained from, I: Kraichnan's equation; II: Burgers' equation, at time $t = 1.6$.

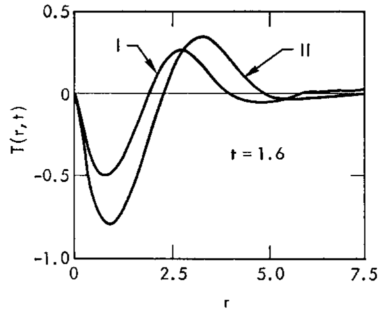

FIG. 3. Nonlinear transport term $T(r, t)$ at time $t = 1.6$, curves I and II being results from Kraichnan's and Burgers' equations, respectively.

and curve II represents

$$T(r, t) = \tfrac{1}{2}[\langle u^2(x, t)u(x + r, t)\rangle - \langle u(x, t)u^2(x + r, t)\rangle],$$

these terms, respectively, describing the nonlinear transport in Eqs. (4) and (2). The amplitude of the nonlinear term in Eq. (4) grows steadily at a greater rate than that in Eq. (2) giving rise to the shift in the correlation curves of Fig. 2 and the greater dissipation rate observed in Fig. 1. This in turn seems attributable to that property of the approximation at infinite Reynolds number of remembering the entire history of the system.

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Autocorrelation Measurements in the Near Wake of Square Cylinders in Turbulent Free Streams

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Eddy-shedding frequencies were measured for square cylinders in cross flow of turbulent air streams of different intensities and scales. The Strouhal number was found to remain almost constant at 0.130 in the Reynolds number range studied.

Although vortex shedding from circular cylinders has been studied extensively, little work appears to have been done with other bluff bodies such as rectangular cylinders. Such work is of theoretical interest in the study of the wake mechanism since the separation point is fixed at the edge so that the wake characteristics should depend only on the flow Reynolds number and the free-stream turbulence.

Recently, Vickery[1] studied the fluctuating lift and drag on a long cylinder of square section in both turbulent and smooth streams. His results indicated that the presence of turbulence in the stream had a marked influence on both the steady and fluctuating forces, especially at low angles of attack (less than 10°). Earlier, Fromm and Harlow made a numerical study of the dynamics[2,3] and also of the heat transfer[4] in the von Kármán wake of a simulated rectangular cylinder. At Reynolds numbers (R) of 100 and 300 (based on cylinder height d, and free-stream velocity V), they obtained Strouhal numbers (S) of 0.119 and 0.137, respectively ($S = fd/V$, in which f is the shedding frequency on one side). These results compare favorably with the value $S = 0.146$ for $R = 1.5 \times 10^5$ given by Fage and Johanson[5].

The objective of the present work was to determine the Strouhal frequency for square cylinders immersed in smooth and turbulent streams, at different angles of attack, and for different flow Reynolds numbers.

The Plexiglass test cylinder spanned the 28-cm square test section of a low-speed wind tunnel. Provision was made to orient the cylinder at any desired angle of attack with the main stream. The wind tunnel was designed to yield a flat velocity profile and a low turbulence level ($< 0.5\%$) at the test section in the absence of turbulence-generating grids. The turbulence level could be changed over 2.5% to 12% by introducing grids upstream of the test section. A hot-wire probe (5 μ in diameter and 1 mm long) was placed in the near wake to detect the velocity fluctuations. The turbulence signal from the constant-temperature hot-wire anemometer was fed to a signal correlator for autocorrelation. For all the time delay ranges used, the time delays were accurate to within 1% and the frequency response flat from 0.36 Hz to well above the highest frequency of interest in this work. The time scale of unsteadiness of the flow was small compared with the averaging time of 20 sec of the computer, e.g., for a 1.25-cm cylinder, it corresponded to about 1000 cycles of vortex shedding at the highest Reynolds number. Although a number of different sizes were used, only data for the 1.25-cm square cylinder are reported in this note.

Figure 1 shows the coordinate system used. Typically, autocorrelograms were obtained with the hot-wire located at $x = 0.95$ cm and $y = 0.95$ cm. In the absence of turbulence grids, the autocorrelograms

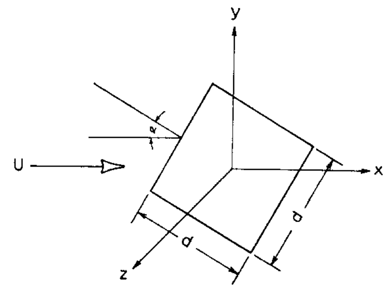

FIG. 1. Coordinate system.

oscillate at the Strouhal frequency. At $\alpha = 0°$, and over the Reynolds number range studied, the Strouhal number was observed to remain almost constant at 0.130.

Introduction of turbulence generators upstream of the cylinder tends to dampen the amplitude of oscillation of the autocorrelograms, but the Strouhal number remains unchanged. Free-stream turbulence was found to increase the turbulence level in the near wake, which is in agreement with the results of Komoda[6] for circular cylinders. Also, the presence of free-stream turbulence suppressed the periodicity in the wake at about $x = 20\,d$ as a consequence of the spectral energy cascade from lower to higher wavenumbers. Visual observation of the turbulence signal on an oscilloscope showed the presence of velocity spikes in the separated shear layer as reported by Bloor[7] and by Hanson, Kozak, and Richardson.[8]

Some autocorrelation measurements were made well outside the wake region where the root-mean-square fluctuation level was equal to the free stream level. Curiously enough, the correlogram indicated a sinusoidal component of Strouhal frequency. A similar observation was made by Maekawa and Mizuno[9] for circular cylinders.

Over the Reynolds number range 1650 to 8000, the Strouhal number was found to remain constant at 0.130. This value is higher than Vickery's value of 0.117 at $\alpha = 0°$ and $R = 1 \times 10^5$. The shedding frequency was found to be slightly higher in turbulent streams at the same wind speed but the difference was of the order of the measurement error.

In order to compensate for any uncertainty in the measurement of the mean speed, the shedding frequencies were also measured for circular cylinders under identical flow conditions. The ratio of the Strouhal frequency for a square cylinder (at $\alpha = 0°$) to that for a circular cylinder of the same projected area to the main stream was approximately constant at a value of 0.62, irrespective of the turbulence level. A large number of wind speeds and turbulence levels were used for making this comparison.

The effect of changing the angle of attack with respect to the mean flow was also studied. Table I

TABLE I. Effect of angle of attack on Strouhal number of a square cylinder (turbulence level < 0.005).

	Strouhal number		
$\alpha°$	$R = 2430$	$R = 3660$	$R = 4700$
0	0.128	0.129	0.128
10	0.128	0.128	0.130
20	0.135	0.141	0.143
30	0.132	0.131	0.132
40	0.126	0.125	0.128
50	0.124	0.125	0.125

shows a typical set of data. The Strouhal number at first increased slightly with α, passed through a maximum at $\alpha = 20°$, and then gradually returned to its value at $\alpha = 0°$. The maximum value for the Strouhal number in all the runs was about 10% higher than the value at $\alpha = 0°$. This again agrees with Vickery's work.

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Modified Discrete Ordinate Approach in Rarefied Gas Dynamics

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An improved discrete ordinate approach involving the Willis integral iteration is tested by application to linearized cylindrical Couette flow. The results agree well with those previously published.

The objective of the work reported here is to test a modification of the usual method of application of the discrete ordinate approach to problems in rarefied gas dynamics. The modification actually involves the use of discrete velocity variables in combination with the Willis' integral iteration scheme. It is hoped that this method will ultimately provide an approach to nonlinear problems which avoids the difficulties involved in a formulation in terms of an integral equation or a set of moment equations, and is numerically more efficient than the usual discrete ordinate method. However, since these methods have not previously been combined, it was felt advisable to test it on a simple problem for which a solution is already available. This will provide not only experience in application but also a check on the accuracy that can be expected. Thus, this note deals with the problem of linearized cylindrical Couette flow.

In particular, we consider the flow between two concentric cylinders, R_1 and R_2, caused by the rotation of the inner cylinder with angular velocity ω_0. The temperatures of the two cylinders are taken to be equal and denoted by T_ω. We nondimensionalize ω_0 by the mean thermal speed and thus define

$$\delta = \frac{\omega_0 R_1}{(2RT_\omega)^{1/2}}.$$

Further, let us choose R_1, $\delta N_0/(2\pi RT_\omega)^{3/2}$, $\delta(2RT_\omega)^{1/2}$, and $4\delta N_0 mRT_\omega/\pi$, where N_0 is the average number density in the annulus, as our scaling factors for distance, molecular velocity distribution function, flow velocity, and shear stress and denote the corresponding dimensionless variables by $\mathbf{r} = (r, \theta, z)$, $\mathbf{c} = (s, \psi, c_z)$, F, $\mathbf{u}$, and τ. (See Ref. 3 for coordinate systems.) The case of interest is $\delta \ll 1$ and we take $F = F_M(1 + \delta f)$ where F_M is now chosen to be the absolute Maxwellian at the outer wall. In these terms the BGK model equation can be written in the form[1,2]

$$s\left(\cos\psi \frac{\partial f}{\partial r} - \frac{\sin\psi}{r}\frac{\partial f}{\partial \psi}\right) = \alpha(2us\sin\psi - f), \qquad (1)$$

where

$$u = \frac{2}{\pi}\int_0^\infty ds s^2 \exp(-s^2)\int_0^\pi d\psi \sin\psi f, \qquad (2)$$

$$\alpha = v_0 R_1/(2RT_\omega)^{1/2},$$

and v_0 is the collision frequency in the undisturbed gas. α is then proportional to an inverse Knudsen number. For convenience, we assume diffuse boundary conditions which, in linearized form, are[2]

$$f = 2s\sin\psi, \qquad r = 1, \qquad |\psi| < \frac{\pi}{2},$$

$$f = 0, \qquad r = b, \qquad |\psi| > \frac{\pi}{2}.$$

The characteristics for Eq. (1) are given by $r\sin\psi = \text{const}$. Thus, regarding u as known and