and

$$\langle D_\alpha \rangle = \frac{\lambda}{\alpha} \cdot \frac{1 - \theta_m}{\theta_m}, \tag{19}$$

where θ_m is the dimensionless temperature.

In case of some polymers where $\Delta H/C_p T_m < 1$ and in dense fluidized beds where $\alpha/\lambda \to 0$ equation (17) reduces to

$$\frac{D}{\sqrt{at}} = 2\{(1 - \theta_m)/(1 + \theta_m - \theta_m^2)^{\frac{1}{2}}\}$$

$$\text{or } \frac{D}{\sqrt{at}} \simeq 2(1 - \theta_m), \qquad \text{for } \theta_m < 0.5, \tag{20}$$

which again is more simple than the following equation given in [4];

$$\frac{D}{\sqrt{at}} = \left\{\frac{12[2\theta + 1 - \sqrt{(2\theta + 1)}]}{2\theta + 5 + \sqrt{(2\theta + 1)}}\right\}^{\frac{1}{2}}.$$

The extensive experimental results given elsewhere [1, 2, 5] verify all of the above theoretical predictions better than with 10 per cent accuracy.

CONCLUSIONS

In this paper an attempt has been made to give a general theory for the process of dip-coating in a fluidized bed. Although the problem is that of finding the solution of a heat transfer problem with moving boundary, unfortunately the presence of the moving boundary makes the equations non-linear, thus preventing one obtaining an analytical solution. However, by integrating the set of equations describing the process it has been possible to find the upper and the lower bounds of the coating thickness as function of time.

The theoretical results have been compared with their literature counterparts and they are found to be more general and more suitable for numerical calculations than the results reported in [4].

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EDDY-SHEDDING FROM A SPHERE IN TURBULENT FREE-STREAMS

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NOMENCLATURE

- D, sphere diameter [cm];
- L, integral scale of turbulence [cm];
- n, frequency of eddy shedding [Hz];
- U, free-stream velocity [cm/s].

Greek symbol

- ν, kinematic viscosity [cm^2/s].

Dimensionless numbers

- Re, Reynolds number, DU/ν;
- St, Strouhal number, nD/U.

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The present note deals with the recent paper by Raithby and Eckert [1] in which they have reported the effect of turbulence intensity, the scale of turbulence, and the position of the support on macroscopic heat transfer from spheres to an air stream in the Reynolds number range 3.6×10^3–5.2×10^4, the Reynolds number being based on the sphere diameter, D, and the free-stream velocity, U.

Hinze [2] showed that $L/D \approx 1{\cdot}2$ for circular cylinders represents the condition for resonance between the energy-containing eddies in the free-stream and the shedding frequency. This value is based on the assumptions that the turbulence field is homogeneous and isotropic and that the cylinder Strouhal number is 0·21. Van der Hegge Zijnen [3] indeed found an 'optimum' value of L/D for which the heat transfer from circular cylinders, for a given turbulence level and Reynolds number, was a maximum. The maximum value was reported as about 1·6. Raithby and Eckert assume the Strouhal frequency for a sphere to be about 10 times that for a cylinder. Application of the resonance hypothesis then yields $L/D = 0{\cdot}12$ as the optimum value for spheres in the appropriate Reynolds number range. Since their data showed that the Nusselt number continued to increase up to at least $D/L = 5{\cdot}0$, there is neither contradiction nor confirmation of the resonance hypothesis in the case of spheres.

As a Strouhal number of 2·1 was intuitively felt to be rather high, some hot-wire measurements were undertaken to determine the shedding frequency for smooth spheres immersed in low-turbulence as well as in highly turbulent free-streams. A literature search revealed that there exists considerable disagreement between results of different workers. Möller [4] (quoted by Raithby and Eckert) reports much higher values of the Strouhal number compared to other workers [5]. For some undisclosed reasons, however, Torobin and Gauvin [5] dismiss the low Strouhal number values as unreliable.

EXPERIMENTAL

A linearized constant temperature hot-wire sensor was placed two diameters downstream and $\frac{1}{4}$ diameter off-centre of a $1\frac{1}{4}$ in. diameter smooth sphere located centrally with a $\frac{1}{16}$ in. diameter cross-flow support in the 11 × 11 in. test section of a wind tunnel. The turbulence level in the clear tunnel was about 0·5 per cent (weakly dependent on the mean velocity). The turbulence level could be varied in the range 2·5–12·5 per cent by introduction of grids, punched plates etc. upstream of the test section. The linearized turbulence signal from the anemometer was fed to a PAR Model 100 Signal Correlator for auto-correlation. Auto-correlation curves were obtained in real time over a variety of time-delay ranges and plotted on an X–Y recorder. Accuracy of the time delay was rated as about 1 per cent. The shedding frequencies were readily computed from the auto-correlograms. The turbulence signal was also recorded on a magnetic tape using an instrumentation grade FM (frequency-modulated) tape recorder for spectral analysis. The shedding frequency was further checked by manually tuning a wave-analyzer with 1 Hz resolution. Figure 1 gives some typical auto-correlograms in the early wake without turbulence-generating grids.

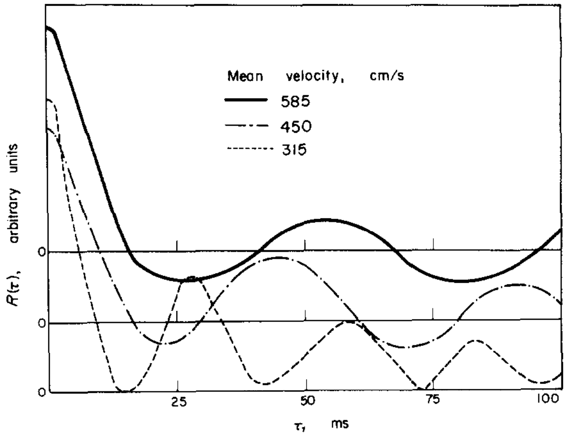

FIG. 1. Auto-correlograms in the near wake of a 3·18 cm diameter sphere; clear tunnel data.

RESULTS AND DISCUSSION

When the turbulence level at the sphere location was about 0·5 per cent, and for a Reynolds number range $5{\cdot}6 \times 10^3$–$11{\cdot}6 \times 10^3$, the Strouhal number was found to remain constant at about 0·20, a value typical of circular cylinders in cross-flow in the same Reynolds number range. Interestingly enough, when the free-stream was made turbulent by introducing turbulence-generating grids, the auto-correlograms decayed without oscillation, thus indicating suppression of eddy-shedding (Fig. 2). Similar experiments with circular and square cylinders under identical conditions of flow showed that the cylinders continue to

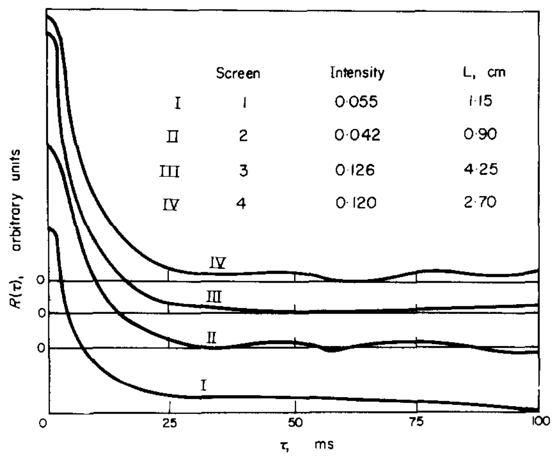

FIG. 2. Auto-correlograms in the near wake when the free-stream is turbulent at a mean velocity of 585 cm/s.

shed eddies without significant change in Strouhal number even when the stream is highly turbulent. There is, therefore, experimental data to support the application of the resonance hypothesis to cylinders. However, if the resonance hypothesis for spheres were correct, transfer should be maximum at about the same value of L/D as for cylinders. Since this effect is now shown by the data reported by Raithby and Eckert, it would appear that for spheres the resonance hypothesis must be rejected. Unfortunately, no spectral data for the free-stream turbulence are available for more realistic estimates of the energy-containing frequencies. More detailed turbulence measurements in the near wake of spheres are required before this uncertainty is finally resolved.

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MASS TRANSFER WITH CHEMICAL REACTION IN A FINITE FALLING FILM

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NOMENCLATURE

c, molar concentration in the liquid;
C, dimensionless molar concentration;
D, molecular diffusivity;
k, specific reaction rate;
m, n, order of reaction with respect to components A, B;
M, dimensionless time, $\pi\theta/2(n + 1)$;
$\bar{N}_{A0}$, average molar flux into liquid film;
$\bar{N}^*_{A0}$, average molar flux for physical absorption alone;
r, rate of reaction;
u, liquid velocity;
u_{max}, liquid velocity at interface;
U, dimensionless liquid velocity, u/u_{max};
x, y, coordinates, Fig. 1;
α, $c_{B0}D_B/\nu c_{Ai}D_A$;
β, $c_{B0}/\nu c_{Ai}$;
γ, angle of inclination to horizontal;
δ, liquid film thickness;
Δ, dimensionless film thickness, $\delta(kc^m_{B0}c^{r-1}_{Ai}/D_A)^{\frac{1}{2}}$;
θ, $kc^m_{B0}c^{r-1}_{Ai}\,(x/u_{max})$;
μ, liquid viscosity;
ν, stoichiometric coefficient;
ρ, liquid density;
ϕ, $\bar{N}_{A0}/\bar{N}^*_{A0}$.

Subscripts
A, B, components A, B;
0, evaluated at time zero;
i, evaluated at the interface.

INTRODUCTION

THE PENETRATION theory of Higbie [1] has been widely applied to unsteady state diffusional problems, with and without chemical reaction. A comprehensive survey of the literature has recently been given by Secor and Beutler [2]. As far as we can ascertain, all the solutions with chemical reaction have been obtained for the case of a semi-infinite body of liquid, although physical absorption into a finite film has been considered [13].

If the liquid were in the form of a finite falling film which is possibly a more realistic situation in chemical engineering, then the rate of mass transfer would be expected to be affected by the finite thickness of the film, since the film would become saturated after a certain distance and diffusion would cease. In this note we consider the problem of diffusion with a generalized chemical reaction into a falling film of finite thickness. The results are compared with those of Brian *et al.* [2, 3] for the semi-infinite case.