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VORTEX SHEDDING FROM SLENDER CYLINDERS OF VARIOUS CROSS SECTIONS

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Vortex shedding frequencies were measured for a number of cylinders of various geometrical shapes by placing a hot-wire sensor in the near wake and autocorrelating the turbulence signal. Strouhal numbers are presented for D-section cylinders with the flat face facing both upstream and downstream, isosceles triangular prisms, axially corrugated and helically grooved circular cylinders. For the triangular prisms effect of angle of attack was also studied. Effect of surface roughness on shedding from circular cylinders was observed to be negligible.

Nomenclature

a, b = sides of triangular prism or lengths to characterize surface corrugations
D = cylinder diameter
D_d = diameter of duct
D_m = mean diameter of test cylinder
D_p = projected width of the test cylinder
L = length of the flat face of D-section cylinder
m = blockage factor or throttling ratio
n = shedding frequency
$R(\tau)$ = autocorrelation function
Re = Reynolds number, UD_p/ν
St = Strouhal number, nD_p/U
U = approach flow velocity
θ = angle of attack to approach flow
ν = kinematic viscosity
τ = time delay (in autocorrelation function)

Introduction

Vortex shedding from slender bodies of circular cross section has been studied quite extensively because of its importance in the design of transmission lines, smoke stacks, alleviation of flow-induced vibration, and noise problems in industrial heat exchangers, etc. [1].[2] Although there has been recent interest in studying the unsteady wake characteristics of noncircular cylinders, e.g., square [2, 3], triangular [4], elliptical [5], conical [6], D-section cylinders [7], etc., the data for these and other noncircular sections of potential engineering interest are not quite so extensive as those for circular cylinders. Mair and Maull [8] have summarized some of the most recent work on the subject of bluff bodies and vortex shedding. Furthermore, effect of surface roughness on shedding has not been studied previously.

Experimental

The test cylinder was mounted horizontally in an $11^1/_2$ in. dia test section of a vertical wind tunnel. Fig. 1 describes the various test models studied. To measure the shedding frequency a normal hot-wire probe (DISA 55A22) was placed about two model widths downstream and a half-model-width away from the cylinder axis. The turbulence signal from the anemometer (DISA 55D01) was fed to a PAR Model 100 Correlator to obtain the correlation function which was plotted directly on a strip-chart recorder. For a stationary time series composed of random fluctuations (due to wake turbulence) superimposed on a periodic component (due to vortex shedding) which is statistically independent of the random fluctuations, the autocorrelation function, $R(\tau)$, is given by

$$R(\tau) = Ke^{-\alpha\tau} + \frac{A^2}{2} \cos(\omega\tau)$$

where α is a measure of the bandwidth of the noise, A is the amplitude of the sine wave, ω is the angular frequency, and τ is the time delay. The eddy-shedding frequency is thus readily obtained from the autocorrellograms. The accuracy of the fre-

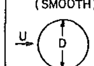
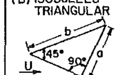
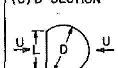
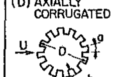
	TEST MODELS	
TYPE	DIMENSIONS	REYNOLDS NUMBER RANGE
(A) CIRCULAR (SMOOTH) 	D = 1/4″, 1/2″, 1″	1,200 - 6,200
(B) ISOSCELES TRIANGULAR 	(a) b θ 1/4″ 11/32″ 1/2″ 11/16″ 0°-90° 1″ 13/8″	870 - 18,000
(C) D-SECTION 	D L 1/4″ 7/32″ 1/2″ 7/16″ 1″ 7/8″	870 - 14,000
(D) AXIALLY CORRUGATED 	D a b 1 1/8″ 1/4″ 1/8″	3,200-14,900
(E) HELICALLY THREADED 	NOMINAL DIAMETER = 1″ STANDARD THREAD : PITCH = 1/8″, 1/16″	3,090-12,900
(F) CIRCULAR (ROUGH)	D = 1.04″ SAND PAPER NO. 80 D = 1.05″ SAND PAPER NO. 60	4,000 - 15,700

Fig. 1 Characteristics of test cylinders used

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[2]Numbers in brackets designate References at end of Note.

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quency measurement is estimated to be $\pm$ 2 percent.

An attempt was made to check the blockage correction proposed by Tsuchiya, et al. [9] for relatively low blockage factors, i.e., $D_p/D_d < 0.10$. The corrected Strouhal number is given by $\mathrm{St} = nD_pm/U$, where the throttling or blockage ratio $m = 1 + 1.25\ (D_p/D_d)$ for $D_p/D_d < 0.35$.

Discussion of Results

Strouhal Numbers for Triangular and *D*-Section Cylinders. Fig. 2 shows the data for *D*-section cylinders corrected for blockage using the formulation of Tsuchiya, et al. [9]. The correction factor does not appear to be satisfactory. Unlike the *D*-section model studied by Feng [7] which was a perfect semicylinder, the present model allows motion of the line of separation with the flat face held downstream. With the flat face upstream, the blockage-corrected Strouhal number was observed to be about 0.14 over the Re range covered which compares favorably with the value 0.135 obtained by Feng. It is noteworthy that this value of St is also close to the values obtained for square cylinders at zero incidence [2, 3], which implies that the shape of the cylinder following separation is not very crucial unless, of course, it interferes with the basic processes of vortex shedding, e.g., a splitter-plate located in the near wake. With the flat face downstream the *D*-section cylinder is "seen" as a circular cylinder by the approach stream resulting in a Strouhal number close to that for a circular cylinder.

For the triangular models St approaches the value for a square cylinder when the largest side faces upstream which confirms the earlier observation concerning the shape of the cylinder following separation. From Figs. 2 and 3 it is clear that the simpler blockage correction proposed by Tsuchiya, et al. [9] fails in this case also and possibly in all cases when the wake may be expected to be much wider than that for a circular cylinder. Parenthetically it is noted that the triangular cylinder sheds very strong vortices when the largest side is upstream suggesting its use as a vortex generator in a vortex shedding flow-meter. The high strength of the periodic component allows use of relatively simpler and inexpensive electronics to detect the frequency because of the high signal-to-noise ratio.

Strouhal Numbers for Roughened and Corrugated Cylinders. The cylinders were made "rough" by wrapping tightly a sand paper around a 1-in. dia circular cylinder. The Strouhal number was observed to remain unaltered at the smooth cylinder value for the two sand paper roughnesses used. Examination of the autocorrelograms indicated existence of strong narrow-band shedding over the entire flow range studied. The sand paper roughness height was too small compared with the cylinder diameter to distinguish between the various diameters that should be used in the definition of St, i.e., the inner, mean, and the outer diameter. In order to distinguish between these, shedding from "extremely rough" cylinders in the form of axially grooved and helically threaded cylinders was studied. For these cylinders shedding occurred at a well-defined frequency yielding a Strouhal number value of 0.205 $\pm$ 0.006 when mean diameter, D_m, was used in the definition of St. (Note: $D_m = D - b/2$, where b is the roughness height.) Clearly, these shapes do not offer any advantages over the normal smooth circular cross sections for transmission wire as far as "galloping" of the wires in strong wind is concerned. Another result of interest in the design of vortex shedding flow-meters is that surface roughness caused by corrosion of the vortex generator would not influence the calibration constant of the meter appreciably.

Effect of Angle of Attack on Shedding From Triangular Cylinders. Table 1 presents some of the data showing the effect of the angle of attack, θ, on shedding from the largest triangular cylinder. Note that the angle is measured from the line bisecting the largest side of the cylinder which faced upstream. No systematic variation of the shedding frequency with θ or Re could be discerned. Unlike in the case of elliptic cylinders (5) use of projected width, D_p, did not produce any systematic correlation for St in terms of θ for Re based on D_p. It was observed from the autocorrelograms that for 15 deg $< \theta <$ 60 deg the second harmonic of the Strouhal frequency becomes quite significant. This contrasts with the case of circular or square cylinders at zero angle of attack which display a pronounced second harmonic only along the wake center line which is influenced by shedding from both sides of the cylinder.

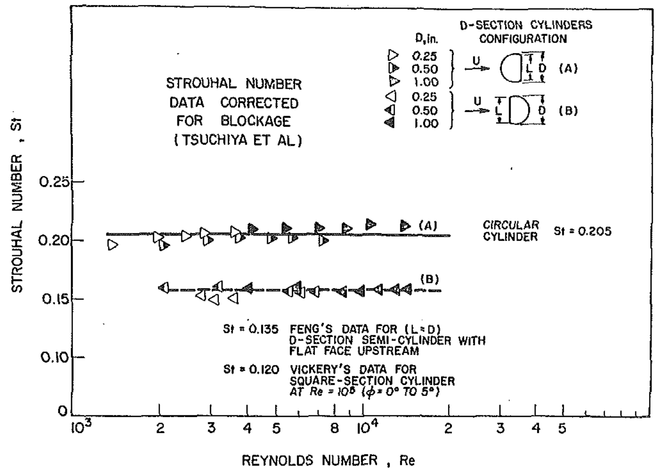

Fig. 2 Blockage-corrected Strouhal number versus Reynolds number characteristic for *D*-section cylinders

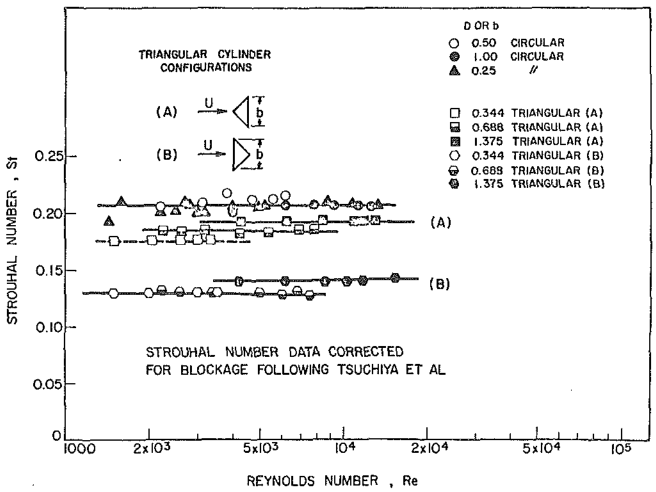

Fig. 3 Blockage-corrected Strouhal number versus Reynolds number plot for triangular cylinders

Table 1 Effect of angle of attack on Strouhal number of triangular cylinders

θ deg.	n_1	n_2	St_1	St_2	$Re_1 \times 10^3$	$Re_2 \times 10^3$
0	29.0	71.8	0.155	0.158	4.0	9.65
15	25.5	72.2	0.160	0.190	4.88	11.75
35	20.0	65.1	0.140	0.190	5.44	13.10
55	25.5	37.8	0.190	0.130	6.50	15.60
75	13.7	35.0	0.130	0.135	7.50	18.10

Note: Subscripts 1 and 2 refer to approach velocities of 10.3 and 24.8 fps respectively. Both St and Re are based on projected widths.

Concluding Remarks

It is suggested that the results of this work could be used in the design of a low pressure loss vortex shedding flowmeter requiring no calibration if accuracy level of ±5 percent is acceptable. A triangular prism is a desirable geometry for the vortex generator because of the strong vortices it generates. Regardless of the cross section chosen for the vortex generator a 7.5 percent lower value is recommended for St when the working fluid is a liquid [10]. When used to meter hot gases (up to 500 deg F) the recent work of Henry [11] suggests that no correction is required for St if the Reynolds number is evaluated at the stream temperature.

Acknowledgments

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Shallow Creeping Flow Round a Corner Under Gravity[1]

A. R. MITCHELL[2] and C. H. TAYLOR[3]

Introduction

THE linearized shallow flow approximation of S. Smith [1][4] and P. Smith [2] is applied to the two-dimensional flow of a very viscous liquid under gravity along a horizontal surface followed by an inclined plane surface (Fig. 1, inset). An ordinary differential equation is solved for the depth of flow. This type of linearized calculation could be used as an initial solution in the computation of a sufficiently viscous and shallow free surface flow. Reasonable agreement is found with some experimental results [3, 4].

Analysis

A coordinate system is chosen with the x-axis parallel to the inclined section of the flow surface at an angle α to the horizontal. The equation of the flow surface is given by $y = f(x)$ and the equation of the free surface $y = h(x)$ is sought. ψ is the stream function, p is the fluid pressure, ρ the fluid density, μ the fluid viscosity, and q the flow per unit width. The well-known exact solution for parallel flow down an inclined plane is used to define $d = (3\mu q/\rho g \sin \alpha)^{1/3}$. The flow is assumed incompressible, two-dimensional and at a sufficiently low Reynold's number to allow inertial terms to be neglected. The Navier-Stokes equations and the boundary conditions determining the flow are given by:

$$p_x = \rho g \sin \alpha + \mu \nabla^2 \psi_y \tag{1}$$

$$p_y = -\rho g \cos \alpha - \mu \nabla^2 \psi_x \tag{2}$$

On $y = f(x)$,

$$\psi = \psi_y = 0 \tag{3}$$

On $y = h(x)$,

$$\psi = q \tag{4}$$

$$\sigma_t = \frac{\mu}{(1 + h_x^2)} \{-4h_x\psi_{xy} + (1 - h_x^2)(\psi_{yy} - \psi_{xx})\} = 0 \tag{5}$$

$$\sigma_n = (p - p_0) - \frac{2\mu}{(1 + h_x^2)} \{h_x^2\psi_{xy} - h_x(\psi_{yy} - \psi_{xx}) - \psi_{xy}\} = 0 \tag{6}$$

Here subscripts x, y denote partial differentiation, p_0 is atmospheric pressure, σ_n, σ_t are normal and tangential free surface stresses. Surface tension effects have been neglected.

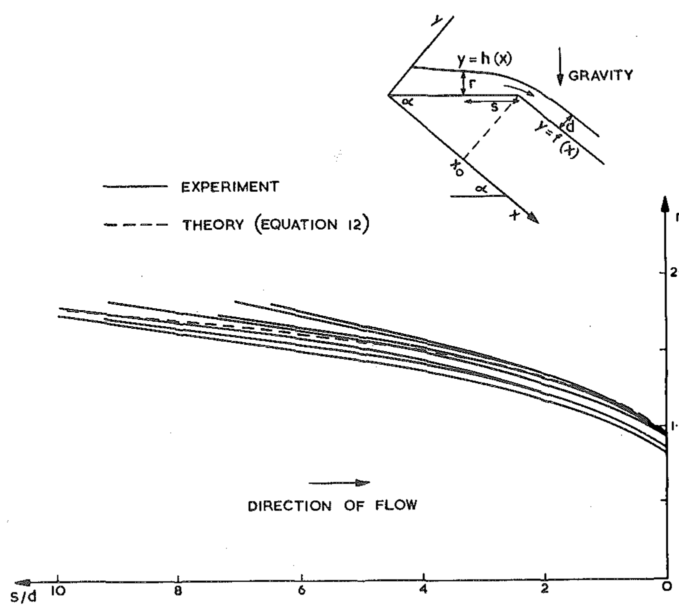

Fig. 1 Plot of nondimensional depth against distance upstream of the corner for seven typical flows from [3]

[1]This note reports a linearized calculation of steady shallow two-dimensiona creeping flow of liquid from a level surface down an inclined plane. Comparison is made with some experimental results.

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[4]Numbers in brackets designate References at end of Note.

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